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# Simple analysis of mixed convection with uniform heat flux

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**Abstract**—When a fluid is forced to flow over a vertical, flat plate, generating uniform heat flux, heat is transferred by forced convection. The temperature difference between the surface and the fluid creates changes in the fluid density, causing natural convection. Therefore forced convection is always coupled with natural convection. In the case of loss of cooling accident (LOCA) in a nuclear reactor, the heat transferred by forced convection can be of the same order of magnitude as that of natural convection, forming a mixed-convection heat transfer mode. When the fluid is forced to flow in the upward direction, we have an assisting mixed convection (AMC), and when the fluid is forced in the opposite direction to the buoyant motion, the condition is opposing mixed convection (OMC). A simple analysis is presented, which evaluates the heat transfer coefficient in AMC and in OMC. The analysis assumes that the hydrodynamic and the thermal boundary layers are the same. It assumes, further, that the velocity profile within the boundary layer is a superposition of pure forced and pure natural convection. A characteristic parameter, $\beta = 3.5U_\infty/\Gamma_L$, is defined which indicates the relative influence of each pure convection mode in the mixed convection phenomena. For AMC a correlation is proposed as follows:

$$\frac{Nu_m}{Re_x^{0.5}} = \left\{[0.483Pr_f^{1/2}]^3 + \left[0.616\frac{(Ra_x^*)^{1/5}}{Re_x^{0.5}}\left(\frac{Pr_f}{0.8+Pr_f}\right)^{1/5}\right]^3\right\}^{1/3}.$$

For OMC, the solution is obtained by dividing the analysis into two regions: the first, a region dominated by forced convection, and the second, a region dominated by natural convection.

## 1. INTRODUCTION

In many of the so-called forced-convection problems encountered in engineering fields, the effect of natural convection is insignificant and can be ignored.

For the cases where the two modes of convection are of the same order of magnitude the coupling effect is important and a mixed convection regime is defined. A practical instance of this occurs in loss of cooling accident (LOCA) events of nuclear reactors where emergency conditions result in a low forced flow superimposed on the natural convection in the reactor core. This is a complex problem, because of the large number of interacting parameters, including the relative direction of the forced and natural convections, the geometry of the arrangement, the flow condition and the boundary conditions. When the fluid is forced in the same direction as the buoyant motion, the regime is an assisting mixed convection (AMC), and when the forced flow is in the opposite direction we have an opposing mixed convection (OMC).

A schematic description of the phenomena is presented in Fig. 1.

For the case of AMC, the early work by McAdams [1] recommended the effect of AMC be ignored and either forced or natural convection used instead, whichever is the higher. The combined effect was presented by Churchill *et al.* [2], suggesting the use of the mathematical form:

$$Y = [1+Z^n]^{1/n} \tag{1}$$

which presents a general expression for a combined effect of two independent continuous phenomena. For the case of AMC, Churchill [3] suggested the use of a correlation based on equation (1) as follows:

$$\frac{Nu_m}{Nu_F} = \left[1+\left(\frac{Nu_N}{Nu_F}\right)^n\right]^{1/n} \tag{2}$$

where $n$ is an exponent, to be selected either from theoretical analysis or according to experimental data. The values of $Nu_N$ and $Nu_F$ are based on correlations for pure convection effect [3].

The analytical–numerical solution by Wilks [4], when put in the form of equation (2), suggests $n = 3$. An earlier work by Acrivos [5] and a full numerical solution by Oosthuizen *et al.* [6] correlated to equation (2) with $n = 4$. On the other hand, Brdlik *et al.* [7] solved analytically only the energy equation in the boundary-layer mixed convection and found $n = 2$.

Other previous solutions [8–11] based on infinite series or complete numerical profiles of the heat transfer coefficient are usually presented in graphs or tables and are difficult to use from a design point of view.

For the cases of OMC, Acrivos [12] in a semi-analytical solution, integrated the conservation equa-

## NOMENCLATURE

| | |
|---|---|
| $B$ | coefficient of volumetric expansion |
| $C_1, C_2, C_3$ | constant of integration (11), (24) and (30), respectively |
| $c_p$ | specific heat capacity |
| $F$ | parametric function |
| $g$ | gravitational acceleration |
| $Gr$ | Grashof number for UWT, $gB\Delta Tx^3/v^2$ |
| $Gr^*$ | Grashof number for UHF, $gBqx^4/(Kv^2)$ |
| $K$ | thermal conductivity |
| $L$ | length |
| $n$ | exponent, equation (1) |
| $Nu$ | Nusselt number |
| $Pr$ | Prandtl number |
| $q$ | heat flux |
| $Ra^*$ | Rayleigh number for UHF, $Gr^*$ $Pr$ |
| $Re$ | Reynolds number |
| $T$ | temperature |
| $U$ | velocity |
| $x,y$ | coordinates |
| $Y$ | dependent function, equation (1) |
| $Z$ | independent function, equation (1). |

Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta$ | dimensionless characteristic parameter, equation (9) |
| $\Gamma$ | normalized velocity caused by natural convection |
| $\delta$ | boundary-layer thickness |
| $v$ | kinematic viscosity. |

Subscripts

| | |
|---|---|
| 0 | wall |
| cr | critical |
| F | forced convection |
| $L$ | characteristic length |
| m | mixed convection |
| N | natural convection |
| $x$ | length dependent |
| $\infty$ | potential flow |
| f | film temperature properties. |
| t | thermal |

tions in the boundary layer, using the Karman–Pohlausen method. His results were presented in the form of $Nu/Re^{0.5} = F(\delta, \delta_t)$ and the function was calculated numerically. In a later work by the same author [5], the previous solution was compared with a complete numerical solution and found to be in disagreement when $Pr > 1$. As a result of that, two correlations were proposed:

(a) For $Pr \rightarrow 0$ the following relation was suggested: $Nu_m^4 = Nu_F^4 - Nu_N^4$, assuming no negative values.
(b) For $Pr \rightarrow \infty$ and above a critical point (which will be discussed later), the following relation was suggested: $Gr_x/Re_x^2 Pr^{1/3} = 1.2$

In a more recent work, Brdlik *et al.* [7] presented an analytical solution for the case of uniform wall temperature. The analysis given in their work, assumed a velocity profile within the boundary layer which is a superposition of natural and forced convection. They came up with one equation for AMC and OMC, by only changing the sign of Grashof number.

The present work evaluates analytically the heat

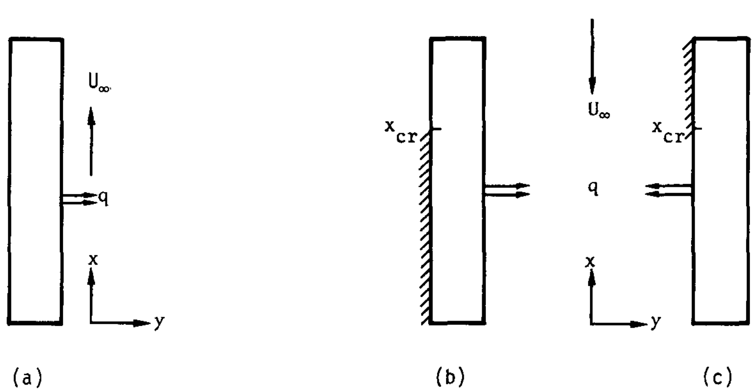


FIG. 1. Mixed convection, different modes: (a) AMC; (b) OMC forced convection dominated; (c) OMC natural convection dominated.

transfer coefficients in mixed convection on a vertical, flat plate with uniform heat flux and in laminar flow conditions. The analysis is based on adopting the concept of superposition of the velocities, and solving the conservation equations. For the case of AMC two approaches were used, whereas for OMC it was assumed that two separate regions exist on the plate, the first where forced convection is dominant and the second where natural convection is dominant.

## 2. THEORETICAL SOLUTION

### 2.1. *Preliminary assumptions*

To simplify the analysis the following was assumed:

(a) Incompressible, steady, laminar flow.
(b) No pressure gradients perpendicular to the direction of flow ($x$-direction).
(c) Negligible heat conduction in the direction of the external field.
(d) Constant potential flow, $U_\infty(x) = U_\infty$, in the external field.
(e) Constant fluid properties, except for the density change with temperature, in the body force term (Boussinesq approximation).

In addition to the above standard assumptions, special assumptions were added as follows:

(f) Mixed convection is characterized by a *common* boundary layer $\delta_m(x)$.
(g) The velocity profile within the boundary layer is a *superposition* of velocity profiles in the two pure convection regimes.
(h) The thermal boundary layer is equal to the hydrodynamic boundary layer (which corresponds to $Pr \cong 1$).

### 2.2. *Basic equations*

For the case of laminar regime and incompressible constant potential flow, with assumptions (a)–(e) cited above, the conservation equations take the following integral forms.

Momentum equation:

$$\frac{d}{dx}\left[\int_0^{\delta_m} U_m(U_m - U_\infty)\,dy\right] = -\nu_f\left(\frac{\partial U_m}{\partial y}\right)_0 + g\beta\int_0^{\delta_m}(T_m - T_\infty)\,dy. \quad (3)$$

energy equation:

$$\frac{d}{dx}\left[\int_0^{\delta_m} U_m(T_\infty - T_m)\,dy\right] = \alpha_f\left(\frac{\partial T_m}{\partial y}\right)_0. \quad (4)$$

As mentioned above, the analysis presented in this work is based on the existence of a common boundary layer $\delta_m(x)$, in which the velocity profile is a superposition of well-known profiles of pure natural and forced convection, as follows:

$$U_m(x) = \Gamma_m\left(\frac{y}{\delta_m}\right)\left(1-\frac{y}{\delta_m}\right)^2 \pm U_\infty\left[\frac{3}{2}\left(\frac{y}{\delta_m}\right) - \frac{1}{2}\left(\frac{y}{\delta_m}\right)^3\right] \quad (5)$$

where the positive sign is for AMC and the negative sign is for OMC.

For pure natural convection and laminar flow regime with uniform heat flux boundary conditions Sparrow *et al.* [13] proposed the following form for the normalized velocity:

$$\Gamma_L = 5.7\frac{\alpha_f}{L}\left(\frac{Pr_f}{0.8+Pr_f}\right)^{2/5}(Ra_L^*)^{2/5} \quad (6)$$

and

$$\Gamma_x = \Gamma_L\left(\frac{x}{L}\right)^{3/5}. \quad (7)$$

The common temperature profile within the boundary layer is assumed to be:

$$T_m - T_\infty = \frac{q\delta_m}{2K_f}\left(1-\frac{y}{\delta_m}\right)^2. \quad (8)$$

A characteristic parameter for mixed-convection phenomena is defined as:

$$\beta = \frac{3.5U_\infty}{\Gamma_L}. \quad (9)$$

Inserting equation (6) into equation [9] to get:

$$\beta = 0.614\,Pr_f^{1/5}(0.8+Pr_f)^{2/5}\frac{Re_L}{(Gr_L^*)^{2/5}}. \quad (10)$$

## 3. ASSISTING MIXED CONVECTION

### 3.1. *First approximation*

In this approximation it is assumed that the value of $\Gamma_m$ in equation (5), is not affected by the forced convection and is equal to that of natural convection, namely, $\Gamma_m = \Gamma_x$.

Using the velocity profile from equation (5) with the positive sign, and the temperature profile from equation (8), with the assumption (h) cited above, and solving the energy equation (4), one obtains:

$$\delta_m^2(\Gamma_x + 3.5U_\infty) = 60\alpha_f x + C_1 \quad (11)$$

at $x = 0$, $\delta_m = 0$ and $C_1 = 0$.

The boundary-layer thickness $\delta_m$ for AMC becomes:

$$\delta_m = \left[\frac{60\alpha_f x}{\Gamma_x + 3.5U_\infty}\right]^{1/2}. \quad (12)$$

The non-dimensional heat transfer coefficient becomes:

$$(Nu_m)_x = 0.483 Re_x^{1/2} Pr_f^{1/2} \left[1 + \frac{1}{\beta}\left(\frac{x}{L}\right)^{3/5}\right]^{1/2} \quad (13)$$

or in another form

$$(Nu_m)_x = \left[0.233 Re_x Pr_f + 0.38\left(\frac{Pr_f}{0.8 + Pr_f}\right)^{2/5} (Gr_x^* Pr_f)^{2/5}\right]^{1/2}. \quad (14)$$

3.2. *The analytical–numerical solution (second approximation)*

In the second approximated analysis it is assumed that $\Gamma_m$ is affected by the forced flow and should be calculated simultaneously with the boundary-layer thickness. Therefore, the momentum and the energy conservation equations are solved by using the preliminary assumptions and by using the velocity and temperature profiles.

Inserting equations (5) and (8) in equations (3) and (4), and performing the integration one obtains:

$$\frac{d}{dx}\left[\delta_m\left(\frac{\Gamma_m^2}{105} + \frac{\Gamma_m U_\infty}{140} - \frac{39}{280}U_\infty^2\right)\right] = -\frac{\nu_f}{\delta_m}\left(\frac{3}{2}U_\infty + \Gamma_m\right) + \frac{gBq\delta_m^2}{6K_f} \quad (15)$$

and

$$\frac{d}{dx}[\delta_m^2(\Gamma_m + 3.5U_\infty)] = 60\alpha_f. \quad (16)$$

Integrating equation (16) with the boundary condition at $x = 0$, $\delta_m = 0$, one obtains:

$$\delta_m^2 = \frac{60\alpha_f x}{\Gamma_m + 3.5U_\infty}. \quad (17)$$

rearranging equation (17) and solving for $\Gamma_m$

$$\Gamma_m = \frac{60\alpha_f x}{\delta_m^2} - 3.5U_\infty. \quad (18)$$

Inserting equation (18) into equation (15) and solving for $d\delta_m/dx$ to obtain

$$\frac{d\delta_m}{dx} = \left[\frac{60(8+7Pr_f)}{Pr_f^2}\left(\frac{x}{\delta_m}\right)^3 - (25 + 14\mathrm{Pr}_f)\frac{Re_x}{Pr_f}\left(\frac{x}{\delta_m}\right) - \frac{7}{6}Gr_x^*\left(\frac{\delta_m}{x}\right)^2\right] \Big/ \left[\frac{720}{\mathrm{Pr}_f^2}\left(\frac{x}{\delta_m}\right)^4 - 25\left(\frac{Re_x}{Pr_f}\right)\left(\frac{x}{\delta_m}\right)^2 + \frac{1}{3}Re_x^2\right] \quad (19)$$

which is a first-order ordinary differential equation.

Equation (19) is solved numerically by the well known fourth-order Runge–Kutta integration routine, with the physical condition of $\Gamma_m > 0$.

From equation (18), this condition becomes:

$$\frac{x}{\delta_m} \geqslant \sqrt{\frac{7}{120} Re_x Pr_f} \quad (20)$$

which defines the starting condition for the numerical integration.

A comparison between the two theoretical results is shown in Fig. 2.

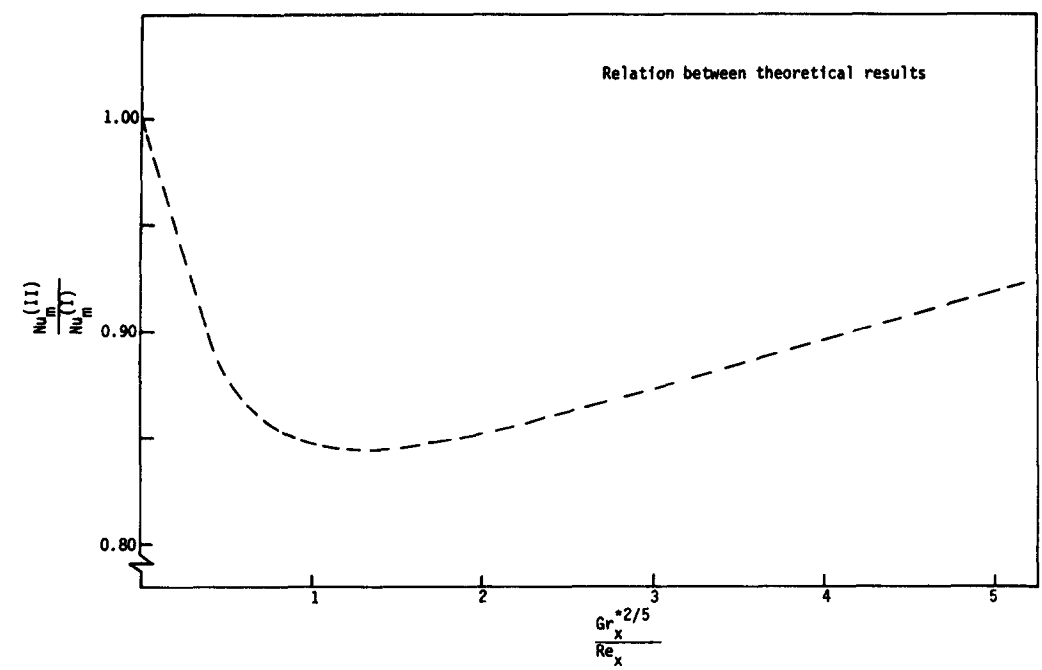


FIG. 2. Comparison between the two theoretical approximations.

## 4. OPPOSING MIXED CONVECTION

For the case of OMC it is assumed that the value of $\Gamma_m$ in equation (5) is not affected by the forced convection, and is equal to that of natural convection, namely $\Gamma_m = \Gamma_x$.

Inserting the velocity profile from equation (5) with the *negative* sign, and the temperature profile from equation (8) with the assumption (h), cited above, and solving the energy equation (4), one obtains:

$$\delta_m^2[\Gamma_x - 3.5U_\infty] = 60\alpha_f x + c_i \tag{21}$$

where $c_i$ is a constant, to be evaluated from the boundary condition. We define a region where forced convection is dominant, when $\Gamma_x - 3.5U_\infty < 0$, and the region where natural convection is dominant, when $\Gamma_x - 3.5U_\infty > 0$. When $\Gamma_x - 3.5U_\infty = 0$, we have a critical point ($x = x_{cr}$), where a transition from one region to another is expected.

The heat transfer coefficient in the uniform heat flux boundary condition is $h_m = k_f/\delta_m$, and in non-dimensional form:

$$Nu_m(x) = \frac{h_m x}{k_f};\quad Nu_F(x) = \frac{h_F x}{k_f};\quad Nu_N(x) = \frac{h_N x}{k_f}. \tag{22}$$

### 4.1. *Dominant forced convection region* (*Fig.* 1*b*)

The forced convection region is defined in the range of $0 < x < x_{cr}$ and $\Gamma_x - 3.5U_\infty < 0$ as mentioned above. At the limit where $x = 0$ also $\Gamma_x = 0$ and the boundary-layer thickness is that of a pure forced convection, namely:

$$(\delta_m^2)_{x=0} = \delta_F^2 = \frac{60\alpha_f L}{3.5U_\infty}. \tag{23}$$

Therefore, the value $c_i$ in equation (21) becomes:

$$c_i = C_2 = -60\alpha_f L. \tag{24}$$

Rearranging equation (21) with the constant $C_2$ from equation (24), and inserting the non-dimensional parameter $\beta$, defined in equation (9), the boundary-layer thickness in the forced convection dominant region becomes:

$$\delta_m(x) = \left\{\frac{60\alpha_f(L-x)}{3.5U_\infty}\left[\frac{1}{1-(1/\beta)(x/L)^{3/5}}\right]\right\}^{1/2}. \tag{25}$$

The requirement of real value for $\delta_m(x)$, provides the upper limit for this region, $x_{cr}$:

$$1 - \frac{1}{\beta}\left(\frac{x}{L}\right)^{3/5} > 0. \tag{26}$$

After rearranging equation (26) the limit becomes:

$$x_{cr} = \beta^{5/3} L. \tag{27}$$

Using equation (22) and the definition of heat transfer coefficient $h_m$, and the boundary-layer thickness $\delta_m$ from equation (25) the normalized heat transfer coefficient of OMC in the forced convection dominant region is:

$$\frac{h_m(x)}{h_F(x)} = \frac{Nu_m(x)}{Nu_F(x)} = \left[1 - \frac{1}{\beta}\left(\frac{x}{L}\right)^{3/5}\right]^{1/2} \tag{28}$$

where

$$h_F(x) = 0.483\left(\frac{K_f}{L-x}\right)\sqrt{\frac{U_\infty(L-x)}{\nu_f}Pr_f}. \tag{29}$$

### 4.2. *Dominant natural convection region* (*Fig.* 1*c*)

In the region of $x_{cr} < x < L$, where natural convection is dominant $\Gamma_x - 3.5U_\infty > 0$ as mentioned above. At the limit $x = x_{cr}$, $\Gamma_x - 3.5U_\infty = 0$, and the condition $\delta_m(x) \neq \infty$ in equation (21) requires:

$$c_i = C_3 = -60\alpha_f x_{cr} = -60\alpha_f L\beta^{5/3}. \tag{30}$$

Rearranging equation (21), inserting the constant $C_3$ from equation (30) and the characteristic parameter $\beta$, the common boundary layer in the dominant natural convection region becomes:

$$\delta_m(x) = \left\{\frac{60\alpha_f L}{\Gamma_L}\left[\frac{(x/L)-\beta^{5/3}}{(x/L)^{3/5}-\beta}\right]\right\}^{1/2}. \tag{31}$$

Using equation (22), the general definition of $h_m$ and the boundary-layer thickness from equation (31), the normalized heat transfer coefficient of OMC in the natural convection dominant region is:

$$\frac{h_m(x)}{h_N(x)} = \frac{Nu_m(x)}{Nu_N(x)} = \left[\frac{(x/L)-\beta(x/L)^{2/5}}{(x/L)-\beta^{5/3}}\right]^{1/2} \tag{32}$$

where

$$h_N(x) = 0.616\frac{K_f}{x}\left(\frac{Pr_f}{0.8+Pr_f}\right)^{1/5}(Ra_x^*)^{1/5}. \tag{33}$$

## 5. DISCUSSION

### 5.1. *Assisting mixed convection*

In order to correlate the results of AMC in this work, the approach of Churchill [3] was adopted, namely:

$$\frac{Nu_m}{Nu_F} = \left[1 + \left(\frac{Nu_N}{Nu_F}\right)^n\right]^{1/n} \quad \text{for } Nu_N < Nu_F \tag{34}$$

and

$$\frac{Nu_m}{Nu_N} = \left[1 + \left(\frac{Nu_F}{Nu_N}\right)^n\right]^{1/n} \quad \text{for } Nu_F < Nu_N. \tag{35}$$

In this approach the two extreme solutions should be defined.

Using equation (14) and letting $Gr_x^* \to 0$ will give the equation for pure forced convection:

$$(Nu_F)_x = 0.483 Re_x^{1/2} Pr_f^{1/2} \tag{36}$$

which is very close to the classical solution of Kays

[14] (for $Pr_f \cong 1$):

$$(Nu_F)_x = 0.453 Re_x^{1/2} Pr_f^{1/3}. \quad (37)$$

Letting $Re_x \to 0$ in equation (14) will give the equation for pure natural convection:

$$(Nu_N)_x = 0.616(Gr_x^* Pr_f)^{1/5} \frac{Pr_f^{1/5}}{(0.8+Pr_f)^{1/5}} \quad (38)$$

which is in agreement with the correlation of Vliet [15] (for $Pr_f \cong 1$):

$$(Nu_N)_x = 0.60(Gr_x^* Pr_f)^{1/5}. \quad (39)$$

By comparing equation (14) with (34) or (35) and using equations (36) and (38), the value of $n$ is immediately determined to be $n = 2$ which is the result of the the first approximation.

The results of the second approximation when using equations (36) and (38) in equation (34) or (35) are best fitted to the curve in which $n = 4$. However, experimental data of Barnea [16] when put in the form of equation (34) or (35) and by using (36) and (38) is correlated best with $n = 3$, as shown in Fig. 3.

Therefore it is suggested to correlate AMC with the following equation:

$$\frac{Nu_m}{Re_x^{0.5}} = \left\{[0.483 Pr_f^{1/2}]^3 + \left[0.616 \frac{(Ra_x^*)^{1/5}}{Re_x^{0.5}} \left(\frac{Pr_f}{0.8+Pr_f}\right)^{1/5}\right]^3\right\}^{1/3}. \quad (40)$$

Equation (40) is presented in Fig. 4. It predicts an increase in heat transfer coefficient for AMC as compared to the pure modes. The maximum discrepancy between the suggested correlation and the first approximation is less than 12% whereas this value is less than 6% for the second approximation.

A similar equation for uniform wall temperature was given by Brdlik *et al.* [7].

*5.2. Opposing mixed convection*

For OMC equations (28) and (32) form a complete solution to the full length of the vertical plate, except at the critical point $x_{cr} = \beta^{5/3} L$, as shown in Fig. 5.

For flow conditions where $\beta$ values are in the range of $0 < \beta < 1$, then $x_{cr} < L$, a critical point occurs within the plate, and a transition from a region of dominant forced convection to a region of dominant natural convection, will take place.

This transition means a change in the direction of the flow. At the dominant forced convection region, the flow near the wall is in the direction of $U_\infty$ (gravity direction), and at $x > x_{cr}$ the flow near the wall is in the upward direction (against gravity).

The analysis of the present work predicts a decrease in heat transfer coefficient towards the critical point, approaching zero value at that point. Mathematically, this means a local sharp increase in wall temperature, however, at that point other mechanisms of heat transfer will occur, resulting in a much lower increase of wall temperature.

In the analytical work of Brdlik *et al.* [7] with uniform wall temperature, the non-dimensional heat

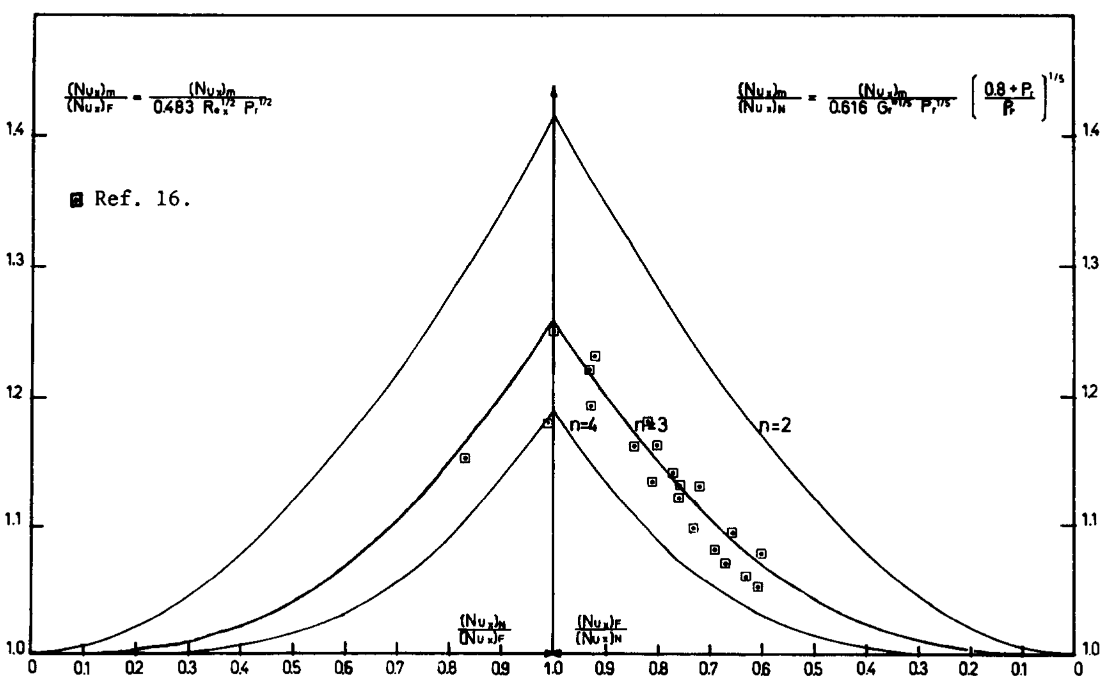


FIG. 3. Experimental data in parametric presentation [equation (2)].

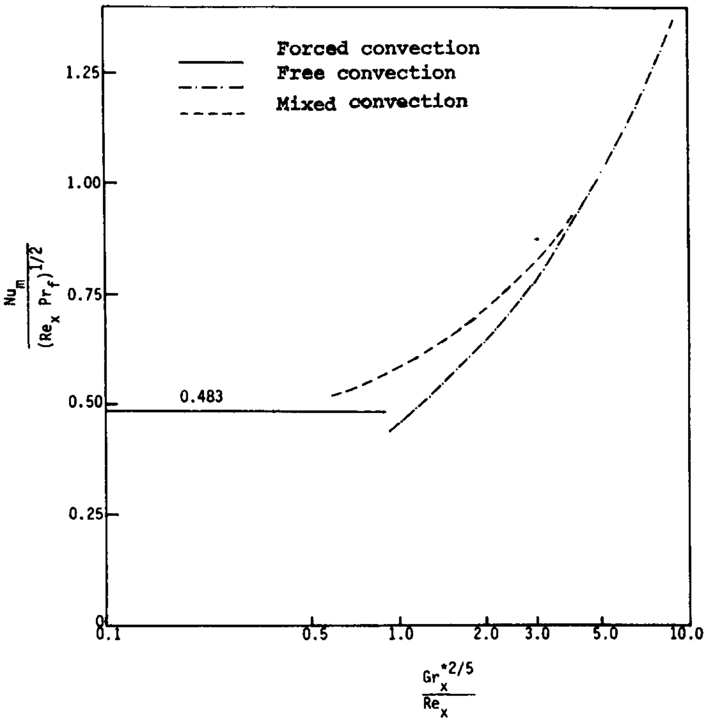


FIG. 4. AMC correlation suggested, equation (40), in comparison with pure convection modes.

transfer coefficient was given as follows:

$$\frac{Nu_x}{Re_x^{0.5}} = \left(\frac{Gr_x}{Re_x^2}\right)^{1/4}$$

$$\times \frac{\{0.346Pr^{1/2}(Re_x^2/Gr_x)^{1/2} \pm 0.51[Pr/(0.952+Pr)]^{1/2}\}}{\{1.015(Re_x^2/Gr_x)^{1/2} \pm [1/(0.952+Pr)]^{1/2}\}^{1/2}} \quad (41)$$

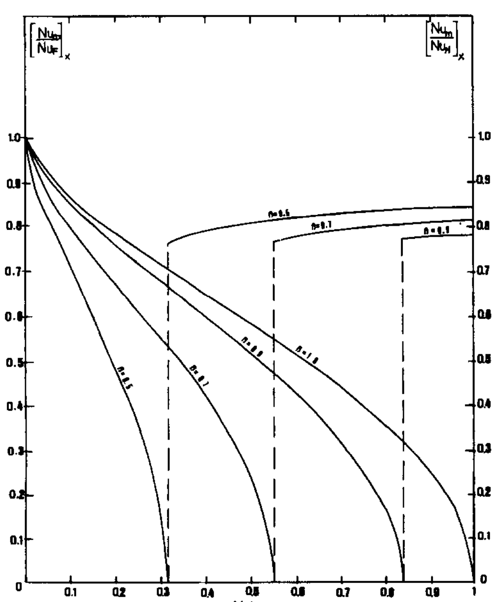


FIG. 5. Heat transfer coefficient in OMC [equations (28) and (32)].

where the positive sign stands for AMC and the negative sign is for OMC. Using the definition of $\beta$ of the present work and rearranging equation (41) for OMC to obtain:

$$Nu_m(x) = 0.343\sqrt{\frac{U_\infty x}{\nu}Pr}\,\frac{[1-(1/\beta)(x/L)^{1/2}]}{[1-2/(3\beta)(x/L)^{1/2}]^{1/2}}. \quad (42)$$

However, by integrating the energy equation for uniform wall temperature and at OMC, one obtains:

$$Nu_m(x) = 0.3416\sqrt{\frac{U_\infty x}{\nu}Pr}$$

$$\times \frac{[1-(1/\beta)(x/L)^{1/2}]}{\{[1-(x/L)]+2/(3\beta)(x/L)^{1/2}\}^{1/2}}. \quad (43)$$

Equation (43) is the correct equation for uniform wall temperature, but both equations (42) and (43) are restricted to the region of dominant forced convection. Moreover, if $\beta < 1$, the solution does not cover the full length of the plate and is limited to $x < \beta^2 L$.

The definition of regions dominated by forced or natural convection, and the analytical solution for each region with uniform heat flux, is the contribution of this work.

## 6. CONCLUSIONS

The simple analysis which is presented in this work provides means for calculating the heat transfer coefficient in laminar external flow in mixed convection. For the case of AMC one simple equation covers the entire length of the plate. However, for OMC two simple equations, depending on whether the region is dominated by forced convection or by natural convection, are needed. The parameter $\beta$ that is used, gives the ratio between the governing characteristic values of each pure mode of convection.

The range of mixed convection is when $0.1 < \beta < 2$. For $\beta < 0.1$, we have pure natural convection, and when $\beta > 2$, we have pure forced convection.

The AMC increases the values of the heat transfer coefficients, however, in OMC this value is decreased in both dominated regions, approaching very low values in the critical point, resulting in an increase of wall temperature.

The importance of such temperature increase, and the uncertainty of the heat transfer mechanism in the transition zone (near $x = x_{cr}$), calls for more experimental work.

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## ANALYSE SIMPLE DE LA CONVECTION MIXTE AVEC FLUX THERMIQUE UNIFORME

**Résumé**—Pour un fluide forcé sur une plaque verticale plane, chauffée à flux thermique uniforme, la différence de température entre la surface et le fluide modifie la densité du fluide et provoque la convection naturelle, d'où un couplage de convection. Dans le cas d'une perte de refroidissement par accident (LOCA) dans un réacteur nucléaire, les deux types de convection peuvent être de même importance. Quand le fluide est forcé dans la direction ascendante, on a une convection mixte assistée (AMC) et en direction opposée c'est le régime de convection mixte contrariée (OMC). On présente une analyse simple qui permet d'evaluer le coefficient de transfert en AMC ou en OMC. Elle suppose que les couches limites dynamiques et thermiques sont les mêmes, aussi que le profil de vitesse dans la couche limite est une superposition de convections pure forcée et pure naturelle. Un paramètre caractéristique $\beta = 3{,}5\, U_\infty/\Gamma$ est défini, lequel indique l'influence relative de chaque mode pur dans le mixage. Pour AMC, on propose une formule

$$\frac{Nu_m}{Re_x^{0,5}} = \left\{[0{,}483 Pr_f^{1/2}]^3 + \left[0{,}616\frac{(Ra_x^*)^{1/5}}{Re_x^{0,5}}\left(\frac{Pr_f}{0{,}8+Pr_f}\right)^{1/5}\right]^3\right\}^{1/3}.$$

Pour OMC, la solution est obtenue en divisant l'analyse en deux régions : la première dominée par la convection forcée et la seconde dominée par la convection naturelle.

## EINFACHE ANALYSE DER MISCH-KONVEKTION BEI GLEICHFÖRMIGER WÄRMESTROMDICHTE

**Zusammenfassung**—Bei erzwungener Fluid-Strömung längs einer vertikalen ebenen Platte, die eine gleichförmige Wärmestromdichte abgibt, wird Wärme durch erzwungene Konvektion übertragen. Die Temperaturdifferenz zwischen Oberfläche und Fluid führt zu einer Dichteänderung im Fluid und damit zu natürlicher Konvektion. Deshalb ist erzwungene Konvektion immer mit natürlicher Konvektion gekoppelt. Im Falle eines Unfalles mit Kühlmittelverlust in einem Kernreaktor kann die durch erzwungene Konvektion übertragene Wärme dieselbe Größenordnung annehmen, wie die durch natürliche Konvektion übertragene und dadurch zur Misch-Konvektion führen. Strömt das Fluid nach oben, so ergibt sich die Bedingung für 'Assisting Mixed Convections' (AMC), und in entgegengesetzer Richtung für 'Opposing Mixed Convection' (OMC). Es wird eine einfache Analyse vorgestellt, welche den Wärmeübergangskoeffizienten für AMC und OMC bestimmt. Für die Analyse wird angenommen, daß die hydrodynamische und die thermische Grenzschicht identisch sind. Es wird ferner angenommen, daß sich das Geschwindigkeitsprofil in die Grenzschicht durch Überlagerung aus reiner erzwungener und reiner natürlicher Konvektion ergibt. Es wird ein charakteristischer Parameter $\beta = 3{,}5U_\infty/\Gamma_L$ definiert, welcher den Einfluß jeder der beiden reinen Konvektionsarten in der Mischform angibt. Für AMC wird folgende Beziehung vorgeschlagen:

$$\frac{Nu_m}{Re_x 0{,}5} = \left\{[0.483\,Pr_f^{1/2}]^3 + \left[0{,}616\,\frac{(Ra_x^*)^{1,5}}{(Re_x)^{0,5}}\left(\frac{Pr_f}{0.8+Pr_f}\right)^{1/5}\right]^3\right\}^{1/3}.$$

Für OMC erhält man die Lösung durch Einteilung in zwei Zonen: In eine erste mit erzwungener und eine zweite mit natürlicher Konvektion.

## ПРОСТОЙ АНАЛИЗ СМЕШАННОЙ КОНВЕКЦИИ С ОДНОРОДНЫМ ПОТОКОМ ТЕПЛА

**Аннотация**—При движении жидкости около вертикальной плоской пластины, генерирующей однородный тепловой поток, происходит передача тепла вынужденной конвекцией. Различие температур поверхности и жидкости вызывает изменения плотности жидкости, вызывая естественную конвекцию. Поэтому вынужденной конвекции всегда сопутствует естественная. В случае аварии с потерей охладителя в ядерном реакторе (АПО), количество тепла, передаваемое вынужденной конвекцией может по порядку величины совпадать с передаваемым естественной конвекцией, создавая смешанно-конвективное течение. Если жидкость движется вверх, имеет место режим усиливающей смешанной конвекции (УСК), при движении жидкости в направлении, противоположном действию подъемной силы, наблюдается режим подавляющей смешанной конвекции (ПСК). Представлен простой анализ, оценивающий коэффициенты теплообмена при УСК и ПСК. В проводимом анализе предполагается, что гидродинамический и тепловой пограничные слои равны, что профиль скорости в пределах пограничного слоя представляет собой суперпозицию чисто вынужденной и чисто естественной конвекций. Найден. характерный параметр $\beta = 3{,}5\,U_\infty/\Gamma_L$, определяющий относительное влияние каждого из режимов чистой конвекции на смешанную. Для УСК предложена следующая зависимость

$$\frac{Nu_m}{Re_x^{0.5}} = \left\{[0{,}483\,Pr_f^{1/2}]^3 + \left[0{,}616\,\frac{(Ra_x^*)^{1.5}}{Re_x^{0.5}}\left(\frac{Pr_f}{0{,}8+Pr_f}\right)^{1.5}\right]^3\right\}^{1\ 3}.$$

Для ПСК решение получено раздельным анализом двух областей: с преобладанием вынужденной и естественной конвекции.